

Directions:

- Unless the question asks for an estimate, give an exact answer (real or complex) in completely reduced form.
- When appropriate, answers should include correct units.
- When specified, you must show work to receive credit for your answers.
- A scientific calculator may be used on the final exam.

Formulas:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, A = Pe^{rt}, A = P \left(1 + \frac{r}{n}\right)^{nt}$$

- Find the difference quotient of f , $\frac{f(x+h)-f(x)}{h}$ where $h \neq 0$, and simplify. [1.4]
 - $f(x) = 7 - 12x$
 - $f(x) = x^2 + 1$
 - $f(x) = x^2 - 2x$
 - $f(x) = 2x^2 + 3x - 6$
- Let $g(x) = 5(x - 23)^2 - 20$. [3.1, 3.2, 3.5, 4.4]
 - Give the coordinates of the vertex.
 - Explain how $g(x)$ is transformed from the graph of $y = x^2$.
 - What are the zeros of $g(x)$?
- Let $g(x) = -\frac{1}{2}(x + 5)^2 + 8$. [3.1, 3.2, 3.5, 4.4]
 - Give the coordinates of the vertex.
 - Explain how $g(x)$ is transformed from the graph of $y = x^2$.
 - What are the zeros of $g(x)$?
- Write the new function $f(x)$ that satisfies the following conditions: $y = |x|$ is reflected with respect to the x-axis, compressed by a factor of $\frac{1}{3}$, shifted to the left three units, and up five units. [3.5]
- Write the new function $f(x)$ that satisfies the following conditions: $y = x^5$ is stretched by a factor of 3, shifted to the right three units, and down five units. [3.5]
- Solve using the most appropriate algebraic method. Show work. [3.2, 3.3]
 - $4x^2 + 2x + 1 = 0$
 - $-2x^2 + 4x - 8 = 0$
 - $7x^2 = -28$
 - $-24 = x^2 - 11x$
 - $25 = x^2 - 8x$
- The cost function for a product is given by $C(n) = n^2 + 12n + 2100$, where n is the number of units produced and sold and $C(n)$ is the cost in dollars. [3.2, 5.2]
 - What is the domain of this function in context of the application?
 - Is the function one-to-one for the domain in part (a)? Explain.
 - Find the inverse of this function for the domain in part (a). Show work. (Hint: Use completing the square).
 - Use the inverse function in part (c) to find how many units are produced and sold if the cost is \$3,220. Show work.
- The cost function for a product is given by $C(n) = n^2 + 6n + 640$, where n is the number of units produced and sold and $C(n)$ is the cost in dollars. [3.2, 5.2]

- a) What is the domain of this function in context of the application?
- b) Is the function one-to-one for the domain in part (a)? Explain.
- c) Find the inverse of this function for the domain in part (a). Show work. (Hint: Use completing the square).
- d) Use the inverse function in part (c) to find how many units are produced and sold if the cost is \$2,312. Show work.

9) Graph by hand. [2.4, 4.2]:

$$a) f(x) = \begin{cases} x^2 - 1, & x < -2 \\ 3x + 2, & -2 \leq x < 2 \\ \sqrt{x + 2}, & x \geq 2 \end{cases}$$

$$b) f(x) = \begin{cases} x - 2, & x \leq 1 \\ \sqrt{3x + 1}, & x > 1 \end{cases}$$

$$c) f(x) = \begin{cases} x^2 - 1, & x < 1 \\ 1 - 2x, & x \geq 1 \end{cases}$$

10) Let $f(x) = 3x - 4$ and $g(x) = \frac{x+4}{3}$. [5.1, 5.2]

- a) Find $f(g(x))$.
- b) Find $g(f(x))$.
- c) Are $f(x)$ and $g(x)$ inverse functions? Explain.

11) Let $f(x) = -8x$ and $g(x) = \frac{1}{8}x$. [5.1, 5.2]

- a) Find $f(g(x))$.
- b) Find $g(f(x))$.
- c) Are $f(x)$ and $g(x)$ inverse functions? Explain.

12) Let $f(x) = 2x + 6$ and $g(x) = \frac{x-6}{2}$. [5.1, 5.2]

- a) Find $f(g(x))$.
- b) Find $g(f(x))$.
- c) Are $f(x)$ and $g(x)$ inverse functions? Explain.

13) Find the inverse of each function. [5.2]

- a) $w(x) = 2x^3 - 5$
- b) $f(x) = \sqrt[5]{x + 7}$

14) Solve each of the following equations algebraically. [2.5, 4.8]

- a) $\sqrt{7x - 28} = \sqrt{x^2 - 4x}$
- b) $|x - 18| = x^2 - 18x$
- c) $|x^2 + 2x - 4| = 4$

15) For each function listed below, identify the type of function and then give the domain and range using interval notation. [1.3, 2.5, 5.4]

- a) $g(x) = 3x^2 - 20$
- b) $h(x) = 3\ln(x)$
- c) $k(x) = |2x + 3| - 8$
- d) $j(x) = 3(4^x)$

- 16) At the end of an advertising campaign, the weekly sales declined. The weekly sales, y (in dollars), are modeled by the equation $y = 12,000(2^{-0.08x})$, where x is the number of weeks after the end of the campaign. [5.3]
- Determine the sales at the end of the campaign.
 - Determine the sales 6 weeks after the end of the campaign.
 - Does the model indicate that sales eventually reach \$0? Explain.
- 17) At the end of an advertising campaign, the weekly sales declined. The weekly sales, y (in dollars), are modeled by the equation $y = 9,000(3^{-0.06x})$, where x is the number of weeks after the end of the campaign. [5.3]
- Determine the sales at the end of the campaign.
 - Determine the sales 9 weeks after the end of the campaign.
 - Does the model indicate that sales eventually reach \$0? Explain.
- 18) Solve each equation algebraically. Show work. When necessary, round answers to four decimal places. [5.6]
- $\log_4(x) = -2$
 - $4 + \log(x) = 10$
 - $e^{(-2x+3)} = 2$
 - $300 = 1200(2^{-0.1x})$
 - $2^{5x-9} = 35$
 - $\ln(-2x + 3) = 10$
 - $2 \ln(x) + 7 = \ln(4x) + 10$
- 19) Rewrite as a single logarithm. [5.5]
- $2 \log(x) + 5 \log(y) - 8 \log(z)$
 - $3 \log_2(m) - 2 \log_2(n) + \log_2(q)$
 - $4 \ln(x) - 7 \ln(y) + 3 \ln(z) - \ln(x)$
- 20) Rewrite as the sum, difference, or product of logarithms and simplify if possible. [5.5]
- $\ln\left(\frac{y^2 e^{3x}}{z^3}\right)$
 - $\log\left(\frac{a^3 c^5}{b^7}\right)$
- 21) Suppose \$ 9,000 is invested is invested for t years at 5.5% interest compounded monthly. [5.3]
- Write an equation that gives the future value, S .
 - Using the model in (a), find the future value of the investment in 4 years.
 - Using the model in (a), find the number of years it will take the investment to double.
- 22) Suppose \$ 6,000 is invested is invested for t years at 7.2% interest compounded quarterly. [5.3]
- Write an equation that gives the future value, S .
 - Using the model in (a), find the future value of the investment in 5 years.
 - Using the model in (a), find the number of years it will take the investment to triple.
- 23) Let $f(x) = 3x^3 + 18x^2 - 12x - 72$. Use this function to answer each question. [4.1, 4.2, 4.4]
- State the degree and leading coefficient of $f(x)$.
 - Find all x such that $f(x) = 0$. Solve algebraically. Show work
 - Describe the end behavior of the graph of $f(x)$.
 - How many turning points does the graph of $f(x)$ have?

24) Let $f(x) = -x^3 + 4x^2 + 9x - 36$. Use this function to answer each question. [4.1, 4.2, 4.4]

- State the degree and leading coefficient of $f(x)$.
- Find all x such that $f(x) = 0$. Solve algebraically. Show work
- Describe the end behavior of the graph of $f(x)$.
- How many turning points does the graph of $f(x)$ have?

25) Solve algebraically. Show work. [4.4]

- $0 = 4x^3 - 4x$
- $x^3 - 15x^2 + 56x = 0$
- $0 = 2x^4 - 3x^3 - 20x^2$
- $x^4 - 3x^3 + 2x^2 = 0$

26) Find the domain, vertical and horizontal asymptotes for each of the following. [4.6]

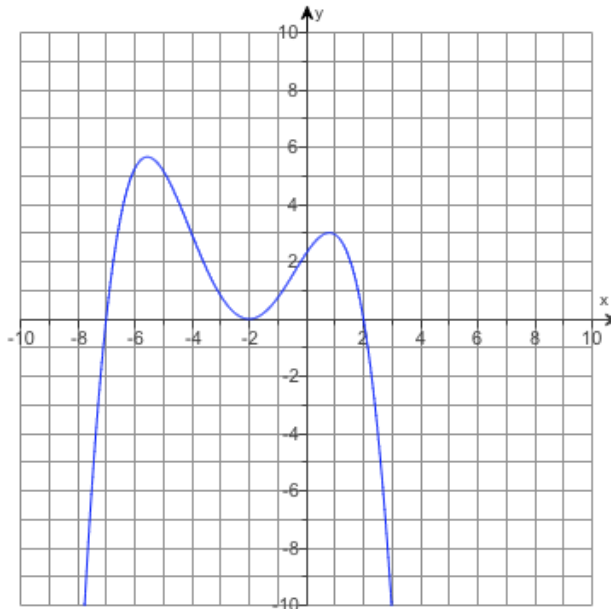
a) $f(x) = \frac{1}{(x-1)^2} + 1$

b) $g(x) = \frac{1-5x}{2x+1}$

c) $f(x) = \frac{x+5}{x^2+7x+10}$

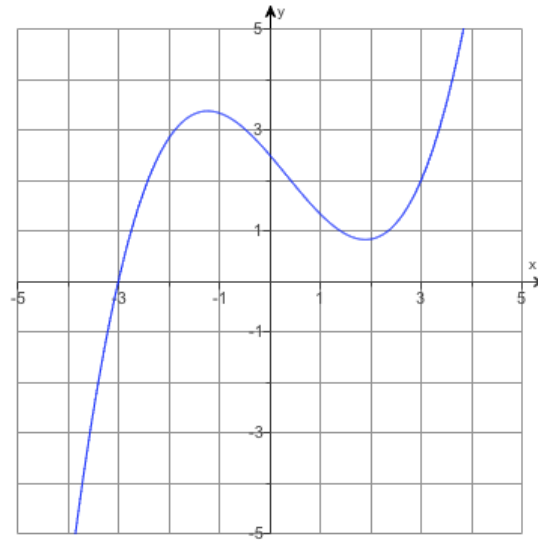
27) Use the graph of the polynomial function f to complete the following. Let a be the leading coefficient of the polynomial $f(x)$. [4.2]

- Determine the number of turning points.
- Estimate the x -intercepts.
- State whether $a > 0$ or $a < 0$.
- Determine the minimum degree of f .



28) Use the graph of the polynomial function f to complete the following. Let a be the leading coefficient of the polynomial $f(x)$. [4.2]

- Determine the number of turning points.
- Estimate the x-intercepts.
- State whether $a > 0$ or $a < 0$.
- Determine the minimum degree of f .



Solutions:

- 1) a) -12
 b) $2x + h$
 c) $2x + h - 2$
 d) $4x + 2h + 3$
- 2) a) $(23, -20)$
 b) Stretched by a factor of 5, shift right 23 units, and shift down 20 units.
 c) $x = 21, x = 25$
- 3) a) $(-5, 8)$
 b) Reflection with respect to the x-axis, compressed by a factor of $\frac{1}{2}$, left 5 units, and shift up 8 units.
 c) $x = -9, x = -1$
- 4) $f(x) = -\frac{1}{3}|x + 3| + 5$
- 5) $f(x) = 3(x - 3)^5 - 5$
- 6) a) $x = \frac{-1 \pm i\sqrt{3}}{4}$
 b) $x = 1 \pm i\sqrt{3}$
 c) $x = \pm 2i$
 d) $x = 8, 3$
 e) $x = 4 \pm \sqrt{41}$
- 7) a) $n \geq 0$ or $[0, \infty)$
 b) Yes. Possible answers for explanation: the domain restrictions allows the function to pass the horizontal line test.
 c) $n(C) = -6 + \sqrt{C - 2064}$
 d) 28 units
- 8) a) $n \geq 0$ or $[0, \infty)$
 b) Yes. Possible answers for explanation: the domain restrictions allows the function to pass the horizontal line test.
 c) $n(C) = -3 + \sqrt{C - 631}$
 d) 38 units
- 9) a) For $x < -2$: decreasing concave up with an open circle at $(-2, 3)$; For $-2 \leq x < 2$: line segment with closed circle at $(-2, -4)$ and open circle at $(2, 8)$; For $x \geq 2$: Increasing, concave down with a closed circle at $(2, 2)$
 b) For $x \leq 1$: increasing line with closed circle at $(1, -1)$; For $x > 1$: increasing, concave down with open circle open circle at $(1, 2)$
 c) For $x < 1$: Increasing, concave up with open circle at $(1, 0)$; For $x \geq 1$: decreasing line with closed circle at $(1, -1)$

- 10) a) $f(g(x)) = x$
 b) $g(f(x)) = x$
 c) Yes; give explanation
- 11) a) $f(g(x)) = -x$
 b) $g(f(x)) = -x$
 c) No; give explanation
- 12) a) $f(g(x)) = x$
 b) $g(f(x)) = x$
 c) Yes; give explanation
- 13) a) $w^{-1}(x) = \sqrt[3]{\frac{x+5}{2}}$
 b) $f^{-1}(x) = x^5 - 7$
- 14) a) $x = 4$ and $x = 7$
 b) $x = -1$ and $x = 18$
 c) $x = -4, 2, -2, 0$
- 15) a) $g(x)$: quadratic, domain : $(-\infty, \infty)$, range: $[-20, \infty)$
 b) $h(x)$: logarithmic, domain : $(0, \infty)$, range: $(-\infty, \infty)$
 c) $k(x)$: absolute value, domain : $(-\infty, \infty)$, range: $[-8, \infty)$
 d) $g(x)$: exponential, domain : $(-\infty, \infty)$, range: $(0, \infty)$
- 16) a) 12,000 dollars
 b) ≈ 8604 dollars
 c) No; the model has a horizontal asymptote at $y=0$
- 17) a) 9,000 dollars
 b) ≈ 4973 dollars
 c) No; the model has a horizontal asymptote at $y=0$
- 18) a) $x = \frac{1}{16}$
 b) $x = 1,000,000$
 c) $x \approx 1.1534$
 d) $x = 20$
 e) $x = 2.8259$
 f) $x \approx -11011.7329$
 g) $x \approx 80.3421$
- 19) a) $\log\left(\frac{x^2y^5}{z^8}\right)$
 b) $\log_2\left(\frac{m^3q}{n^2}\right)$
 c) $\ln\left(\frac{x^3z^3}{y^7}\right)$

- 20) a) $2 \ln(y) + 3x - 3 \ln(z)$
b) $3 \log(a) - 7 \log(b) + 5 \log(c)$
- 21) a) $S = 9000 \left(1 + \frac{0.055}{12}\right)^{12t}$
b) $\approx 11,209.06$ dollars
c) $t \approx 12.6$ years
- 22) a) $S = 6000 \left(1 + \frac{0.072}{4}\right)^{4t}$
b) $\approx 8,572.49$ dollars
c) $t \approx 15.4$ years
- 23) a) *degree* $n = 3$; *leading coefficient* $a = 3$
b) $x = -6, -2, 2$
c) $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$, $f(x) \rightarrow \infty$ as $x \rightarrow \infty$
d) 2 turning points
- 24) a) *degree* $n = 3$; *leading coefficient* $a = 1$
b) $x = 4, -3, 3$
c) $f(x) \rightarrow \infty$ as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$ as $x \rightarrow \infty$.
d) 2 turning points
- 25) a) $x = -1, 0, 1$
b) $x = 0, 7, 8$
c) $x = 0, \frac{-5}{2}, 4$
d) $x = 0, 1, 2$
- 26) a) Domain: $(-\infty, 1) \cup (1, \infty)$; V.A.: $x = 1$; H.A.: $y = 1$
b) Domain: $(-\infty, \frac{-1}{2}) \cup (\frac{-1}{2}, \infty)$; V.A.: $x = \frac{-1}{2}$; H.A.: $y = \frac{-5}{2}$
c) Domain: $(-\infty, -5) \cup (-5, -2) \cup (-2, \infty)$; V.A.: $x = -2$, (*Hole at* $x = -5$); H.A.: $y = 0$
- 27) a) 3; b) $(-7,0), (-2,0), (2,0)$; c) $a < 0$; d) The minimum degree of f is 4.
- 28) a) 2; b) $(-3,0)$; c) $a > 0$; d) The minimum degree of f is 3.